# Electronic Structure of Carbonium Ions. <br> Methyl and Ethyl Cations ${ }^{1 a}$ 

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#### Abstract

Ab initio wave functions have been obtained for $\mathrm{CH}_{3}+$ and $\mathrm{CH}_{4}$ and for $\mathrm{CH}_{3} \mathrm{CH}_{2}+$ and $\mathrm{CH}_{3} \mathrm{CH}_{3}$ by means of the LC (Hartree-Fock)-AO-MO-SCF scheme. According to the calculated energy differences between the cations and their hydrocarbon precursors, the ethyl cation has an "extra stabilization" of $18 \mathrm{kcal} / \mathrm{mole}$ over the methyl cation, in rather good agreement with experimental results. Population analyses, energy term separation, and correlation diagrams provide insight into the electronic structure of carbonium ions in general and lend a new explanation for the stabilizing effect of methyl substitution at a positive center.


TThe ethyl cation is an obvious and convenient model for larger alkyl cations because it is the simplest system in which appears the effect of alkyl substitution onto a carbonium ion center. Like most carbonium ions, however, the ethyl cation is far too reactive to allow thorough experimental investigation. Rigorous quantum theory therefore provides the most reliable and detailed source of information on the forces governing the structures and stabilities of the ethyl cation and other simple alkyl cations.

Numerous $a b$ initio ${ }^{1-5}$ and semiempirical ${ }^{6-10}$ calculations on alkyl cations other than the methyl cation have appeared recently. Those dealing explicitly with the ethyl cation ${ }^{1,5,6-10}$ fall into two general categories. The first comprises the relatively crude $\pi$ electron calculations ${ }^{7}$ which attempt to evaluate the energy associated with hyperconjugation. In these calculations

the three methyl hydrogens are treated as a pseudoatom triply bound to the methyl carbon. Such treatments explicitly take into account only those orbitals or linear
(1) (a) Supported in part by the Power Branch of the Office of Naval Research, Contract Number N00014-67-A-0151-0016; (b) J. E. Williams, Jr., V. Buss, L, C. Allen, P. v. R. Schleyer, W. A. Lathan, W. A. Hehre, and J. A. Pople, J. Amer. Chem. Soc., 92, 2141 (1970); (c) G. V. Pfeiffer and J. G. Jewett, ibid., 92, 2143 (1970).
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(10) J. J. Dannenberg and T. D. Berke, 158th National Meeting of the American Chemical Society, New York, N. Y., Sept 1969, Abstract PHYS 163.
combinations of orbitals of the proper symmetry to mix in with the cation pseudo- $\pi$ system. The energetic consequences of such mixing can be taken care of only ad hoc, through the use of adjustable parameters, because these methods manifestly cannot adequately reproduce the proper balance of one-electron effects (nuclear-electron attraction and electron kinetic energy) vs. two-electron effects (electron-electron repulsion and exchange energies). The importance of hyperconjugation is thus consistently overestimated ${ }^{11}$ and resulting calculated charge densities disagree greatly with those found using more refined and reliable techniques.

Calculations on the ethyl cation in the second category include the $\sigma$ electrons and are used to estimate quantities other than resonance energies. Both $a b$ initio and semiempirical schemes have been investigated, and it is worthwhile noting that even the twoelectron containing semiempirical schemes can yield results opposite to $a b$ initio wave functions for some important cases. Thus ab initio calculations ${ }^{2}$ for the ethyl and vinyl cations show them to be more stable than the isomeric bridged structures, protonated ethylene and protonated acetylene, respectively, while semiempirical calculations give the reverse. ${ }^{9,10}$ Small basis set $a b$ initio wave functions also can be misleading; thus Preuss, et al. ${ }^{5}$ found the ethyl cation and protonated ethylene structures to be of equal energy ( -77.779 hartrees) due to an inadequate representation of the hydrogens.

Method of Calculation. Ab initio calculations, in the Roothaan ${ }^{13}$ LCAO-MO-SCF approximation, are reported in detail for the methyl cation 3 and the ethyl cation 4, for their parent hydrocarbons, methane 1 and ethane 2, for the $\pi$-localized ethyl cation 5 , and for protonated ethylene 6 (Figures 1 and 3). Ab initio results for several of these structures have been summarized in previous reports. ${ }^{1 \mathrm{~b}, 2}$ Structure 5 differs from 4 only in that the empty $p$ orbital on the $\mathrm{CH}_{2}{ }^{+}$ group of the latter has been left out of the calculation. The computations for $\mathbf{1 - 3}$ done earlier in this laboratory ${ }^{14-16}$ were redone with the more flexible hydrogen basis used for the other structures.
(11) Extensive delocalization through an alkyl cation pseudo- $\pi$ system should give rise to a long wavelength transition (at $c a .290 \mathrm{~m} \mu$ ) observable in the electronic spectra of these cations (S. F. Mason, Quart. Rev., Chem. Soc., 15, 339 (1961)), but only end absorption is actually found. 12
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Table I. Structures for Species 1-6 ${ }^{a}$

| No. | Structure | $R(\mathrm{CC})^{b}$ | $\Varangle \mathrm{HC}^{+} \mathrm{H}^{c}$ | $\Varangle \mathrm{HCCH}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Methane |  |  |  |
| 2 | Ethane | 1.534 |  | $60,60,60^{d}$ <br> (staggered) |
| 3 | Methyl cation |  | 120 |  |
| 4 | Ethyl cation | 1.48 | 120 | 90, $90^{\circ}$ |
| 5 | Ethyl cation, $\pi$ localized | 1.48 | 120 | 90, 90 |
| 6 | Protonated ethylene | $1.46{ }^{\prime}$ |  | Ethylene moiety planar ${ }^{a}$ |

${ }^{a}$ Reasons for choice of values of structural parameters for 1 and 6 are detailed in ref 2 ; geometries for all other species are idealized or derived from 4. ${ }^{b}$ All lengths in angströms; for tetragonal carbon $R(\mathrm{CH})=1.096$; for trigonal carbon $R\left(\mathrm{C}^{+} \mathrm{H}\right)=1.084$ with the exception of the bridge hydrogen in 6. ${ }^{\circ}$ Angles in degrees; $\mathrm{CH}_{3}$ groups are kept tetrahedral. ${ }^{d}$ Smallest dihedral angles in this conformation. ${ }^{e}$ Dihedral angles between hydrogens of $\mathrm{CH}_{2}{ }^{+}$and unique methyl hydrogen. $f R\left(\mathrm{CH}_{\text {bridge }}\right)=1.33 .{ }^{a}$

Role of the Methyl Group in Stabilizing the Ethyl Cation. The stabilization of carbonium ions resulting from the substitution of methyl groups for hydrogens attached to the positive center has long been known by organic chemists. For instance, experimental heats of formation show the ethyl cation to be more stable than the methyl cation by an amount $31^{21}-42^{22} \mathrm{kcal} / \mathrm{mol}$ greater than the difference in the heat of formation ${ }^{23}$ between ethane and methane. This "extra" methyl stabilization energy in alkyl cations may be rationalized using disarmingly straightforward logic. The most obvious difference between a highly reactive, and in this sense, unstable cation such as 4 and its unreactive, neutral precursor 2 , is the presence in 4 of an electrondeficient center. The instability associated with such a positively charged center can be offset by neutralizing the charge, or at least dispersing it throughout the

Table II. Ab Initio Total Energies and Energies Components for Ethyl and Methyl Cation and Their Neutral Hydrocarbon Precursors ${ }^{a}$

| No. | Species | $E_{\mathrm{t}}{ }^{b}$ | $V_{\mathrm{nn}^{b}}$ | $V_{e \mathrm{e}}{ }^{b}$ | $T^{b}$ | $V_{\mathrm{ne}}{ }^{b}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Methane | -40.1788 | 13.3738 | 25.9622 | 40.1788 | -119.6982 |
| 2 | Ethane | -79.1851 | 42.2306 | 67.3506 | 79.1851 | -267.9515 |
| 3 | Methyl cation | -39.2072 | 9.6478 | 19.0193 | 39.2074 | -107.0819 |
| 4 | Ethyl cation | -78.2426 | 37.1830 | 57.1808 | 78.2425 | -250.8490 |

${ }^{a}$ All energies in hartrees ( 1 hartree $=627.502 \mathrm{kcal} / \mathrm{mol}$ ) and scaled to satisfy the virial theorem. ${ }^{b} E_{t}=$ total energy relative to electrons and nuclei all at infinitive separations; $V_{\mathrm{nn}}=$ nuclear repulsion energy; $V_{\mathrm{ee}}=$ electron-electron interaction inergy; $T=$ electron kinetic energy; $V_{\mathrm{ne}}=$ nuclear-electron attraction energy.

Table III. Ab Intitio Total Energies and Energy Components for Cations 5 and $6^{*}$

| No. | Species | $E_{\mathrm{t}}$ | $V_{\mathrm{nn}}$ | $V_{\text {ee }}$ | $T$ | $V_{\text {ne }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | $\pi$ Localized ethyl cation, 90, 90 | -78.2235 | 37.1562 | 57.1161 | 78.2235 | -250.7194 |
|  | $\begin{aligned} & \text { Difference } \\ & \mathbf{4 \rightarrow 5} \end{aligned}$ | +11.9 | $-16.7$ | -40.4 | -11.9 | +81.0 |
| 6 | Protonated ethylene | -78.2285 | 37.7902 | 57.5040 | 78.2285 | -251.7512 |
|  | Difference $4 \rightarrow 6$ | +8.8 | $+379.0$ | +201.8 | -8.8 | -564.0 |

${ }^{a}$ Absolute energies in hartrees and scaled to satisfy the virial theorem. Energy differences in kilocalories per mole.

The basis set for all calculations consisted of Whitten's ${ }^{17}$ three s-type groups (total of ten Gaussians) and Huzinaga's ${ }^{18}$ five Gaussian $p$ orbitals on carbon, and Whitten's five $s$-type Gaussians grouped into a fourcomponent and a long-range one-component group on each hydrogen, both groups scaled with $\eta^{2}=1.44$. $A b$ initio calculations with such a basis give structural parameters to within a few per cent ${ }^{19}$ and threefold barriers to within $\pm 0.4 \mathrm{kcal} / \mathrm{mol} .{ }^{20}$

Structures for the species $\mathbf{1 - 6}$ are as reported previously and are summarized in Table I. Population analyses are given in Figures 1 and 3, total energies and energy components in Table II and III, and the nuclear coordinates and the molecular orbital energies in Table IV.
(14) W. H. Fink and L. C. Allen, to be published.
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(19) L. C. Allen in P. O. Lowdin, "Quantum Theory of Atoms, Molecules, and the Solid State: A Tribute to John C. Slater," Academic Press, New York, N. Y., 1966.
(20) L. C. Allen, Chem. Phys. Lett., 2, 597 (1968).
molecular framework. The stability of 4 relative to 3 therefore seems to imply that methyl groups are better electron suppliers than hydrogens. This conclusion is widely accepted. ${ }^{24-26}$

This argument, however, is seriously flawed. The positive center in 4 is only a part of the molecule. Increasing the electron density at such an electrondeficient site will undoubtedly increase the nuclear-
(21) J. L. Franklin in "Carbonium Ions," Vol. I, G. Olah and P. v. R. Schleyer, Ed., Wiley-Interscience, New York, N. Y., 1969; D. Bethell and V. Gold, "Carbonium Ions: An Introduction," Academic Press, New York, N. Y., 1967.
(22) Calculated using data of J. A. Kerr, Chem. Rev., 66, 465 (1966), and of C. E. Melton and W. H. Hamill, J. Chem. Phys., 41, 3464 (1964), and F. A. Elder, C. Giese, B. Steiner, and M. Inghram, ibid., 36, 3292 (1962). The agreement in the ionization potential for $\mathrm{C}_{2} \mathrm{H}_{5}$ in the last two works, the former using electron impact, the latter photoionization, makes the higher value of $42 \mathrm{kcal} / \mathrm{mole}$ more probable.
(23) American Petroleum Institute, Project 44, "Selected Values of the Thermodynamic Properties of Hydrocarbons," Carnegie Institute of Technology, Pittsburgh, Pa., 1952 onward.
(24) J. D. Roberts and M. C. Caserio, "Basic Principles of Organic Chemistry," W. A. Benjamin, Inc., New York, N. Y., 1964; R. W. Taft, Jr., and I. C. Lewis, Tetrahedron, 5, 210 (1959).
(25) E. S. Gould in "Mechanism and Structure in Organic Chemistry," Holt Rinehart, and Winston, New York, N. Y., 1959, Chapter 7. (26) M. J. S. Dewar, "Hyperconjugation," Ronald Press Co., New York, N. Y., 1962.
electron attractions at that site and thus lower its energy, but this does not necessarily mean that the energy of the cation as a whole will decrease. ${ }^{27}$ While the question of the electron-releasing ability of methyl relative to hydrogen is still an ambiguous one, there is a more effective way of stabilizing a cationic center which does not depend on the electron-donating ability of the methyl group.

It is convenient in discussing the ethyl cation to make the usual separation of effects into $\sigma v s . \pi$ effects. The $\pi$ effects-including any that might be called $\pi$ inductive effects-we loosely term hyperconjugative and discuss in the next section. Throughout this paper we refer to the "empty" $p$ orbitals of $\mathbf{3 - 5}$ as $p_{\pi}$ orbitals, comprising, with the parallel $p_{\pi}$ orbital of the methyl group, the "pseudo- $\pi$ " or $\pi_{z}$ system. The $\pi$-type $p$ orbitals perpendicular to the $\pi$ system are the $\mathrm{p}_{v}$ orbitals and make up the $\pi_{y}$ system.

From Table II the difference in reaction energies between 1 and 2, which equals the "extra" methyl

$$
\begin{gather*}
\mathrm{CH}_{4} \longrightarrow \mathrm{CH}_{3}^{+}+\mathrm{H}^{-}  \tag{1}\\
\mathrm{C}_{2} \mathrm{H}_{6} \longrightarrow \mathrm{C}_{2} \mathrm{H}_{5}^{+}+\mathrm{H}^{-} \tag{2}
\end{gather*}
$$

stabilization energy for the ethyl cation, is found to be $18 \mathrm{kcal} / \mathrm{mol}$. This differs from the experimental values mentioned above for two major reasons. The four species 1-4 contain different numbers of atoms and bonds and thus their energies are at different distances from their respective true Hartree-Fock energies, and also zero-point energy changes are not included in our $a b$ initio calculations. Estimates of the zero-point energy difference between 1 and 2 from the values of Pitzer and Catalano, ${ }^{28}$ and of the corresponding difference between 3 and 4 from the (incomplete) set of calculated frequencies for $\left(\mathrm{CH}_{3}\right)_{3} \mathrm{C}^{+}$and $\left(\mathrm{CH}_{3}\right)_{2} \mathrm{CH}^{+}$given by Olah, et al., ${ }^{29}$ indicate that correcting the experimental stabilization energy for zero-point energies will lower it significantly-probably by $15 \mathrm{kcal} / \mathrm{mol}$. Electron correlation (intramolecular van der Waals) energy changes-which are outside the scope of the HartreeFock model-may be also fairly large for each of the two reactions (1 and 2). It is very likely, however, that the difference in correlation energies between the two reactions will not be very significant-no more than 1 or $2 \mathrm{kcal} / \mathrm{mol}$.

To find the origin of the extra methyl stabilization, we turn first to the population analyses presented in Figure 1, which might help us decide as to the electrondonating ability of $\mathrm{CH}_{3}$ relative to hydrogen. Unfortunately, because the electrons are divided between the different nuclei in an artificial way, ${ }^{30}$ the accuracy of the electronic distribution as represented by population analysis does not necessarily go hand in hand with the quality of other calculated values, such as the total energy. And, in fact, with an inferior basis set the distribution of electrons might yield more reasonable atomic charges than one which results from the highly sophisticated basis we are using in the present

[^0]

1


3


2

4

Figure 1. Ab initio formal atomic charges and overlap populations (in parentheses) for the methyl and ethyl cations and their parent hydrocarbons.
calculation. Thus a minimum Slater-type basis set (with optimized exponents) showed ${ }^{31 a}$ that for methane the hydrogens had a positive charge each of 0.021 , leaving an excess of 0.084 electron on the carbon atom. This is clearly more realistic than the population analysis of methane shown in Figure 1 which renders the carbon negative by 1.072 electrons.

The positive atomic charge on the hydrogens gives the amount by which hydrogen is a better electron donor than methyl, for in ethane, because of its symmetry, the charge on each methyl group is zero. This value, 0.268 in our calculation and 0.021 in Pitzer's, seems to indicate that in neutral molecules the methyl group is indeed electron-withdrawing relative to hydrogen.

Turning to the cationic compounds, we find that in $\mathrm{CH}_{3}{ }^{+}$the hydrogens have transferred 0.515 electron each to the positive center, whereas in the ethyl cation the methyl group has donated only 0.323 electron. Again the methyl group is losing less electrons than is hydrogen, but we note that the difference now is only 0.192 electron as compared to 0.268 in the neutral precursors. Thus it seems that in carbonium ions in which hyperconjugation is an important factor, the methyl group is a better electron donor than in hydrocarbons, and since the value of 0.268 is probably highly exaggerated, we conclude that the methyl group, if bonded directly to a cationic center, might even be a better electron donor than hydrogen. However, the atomic population based analysis presented here is not really convincing because, not only is it a very rough and uncertain estimate of the charge distribution as indicated, but so far we have not taken into account what is happening in other parts of the molecular system as well. A more complete understanding of the problem follows from the reasoning developed below.

The experimental CH homolysis of ethane is only $6 \pm 2 \mathrm{kcal} / \mathrm{mol}$ lower than that of methane, ${ }^{21}$ so the extra methyl stabilization arises as a result of changes occurring during ionization of the radicals derived from 1 and 2. Furthermore, since we expect the relief in nuclear repulsion (Table II) to occur mainly in the radical formation step, in which the tetrahedral carbon changes to trigonal coordination, ${ }^{2,31 \mathrm{~b}}$ the effects we are looking for can be found only in the electronic energy changes for reactions 1 and 2 . In order to examine

[^1]

Figure 2. Correlation diagram of the MO partitioned energies $e_{\mathrm{i}}$ for reactions 1 and 2 .
energetic changes in the MO's, we use the partitioned total energy quantities, $e_{i}$, rather than the ionization potential related eigenvalues of the Hartree-Fock equations, $\epsilon_{i}$. The $e_{i}$ add up to the total electronic energy and the component composition of the two types of one-electron energies are contrasted in the following definitions

$$
\begin{gathered}
\epsilon_{i}=V_{\mathrm{ne}}(i)+T(i)+V_{\mathrm{ee}}(i) \\
e_{i}=V_{\mathrm{ne}}(i)+T(i)+1 / 2 V_{\mathrm{ee}}(i)
\end{gathered}
$$

The $e_{i}$ for 1-4 are used to construct the valence-electron correlation diagram ${ }^{32}$ of Figure 2.

Correlations between methane and ethane connect bonding orbitals with a high percentage of a given atomic orbital from the carbon of methane with the corresponding linear combination of carbon orbitals on ethane. These are either a symmetric (labeled $\mathbf{S}$ ) or antisymmetric (labeled A). Correlations for the RH $\rightarrow$ $\mathrm{R}^{+}$transformations are labeled with the symmetry of the carbon orbital characterizing the MO's being correlated: $\mathrm{s}=2 \mathrm{~s}$ (actually the third carbon s -type group ${ }^{17}$ ), $\mathrm{x}=$ $2 \mathrm{p}_{\sigma}, \mathrm{y}=2 \mathrm{p}_{y}, \mathrm{z}=2 \mathrm{p}_{\pi}$.

Figure 2 shows the lowering in $e_{i}$ for each occupied MO of $\mathbf{1}$ or 2 relative to the corresponding MO of $\mathbf{3}$ or 4. The difference between the sums of these energy lowerings for $1-3$ vs. 2-4 accounts for the "extra"
(32) For changes involving only angular coordinates, we may draw correlation diagrams-such as Walsh diagrams [A. D. Walsh, J. Chem. Soc., 2260 (1953)]-using the molecular orbital energies $\epsilon_{1}$, provided the bonding in the systems investigated is not ionic [S. D. Peyerimhoff, R. J. Buenker, and L. C. Allen, J. Chem. Phys, 45, 734 (1966)]. This is not true in general for reactions like 1 and 2 , which represent bond-length changes and involve large amounts of charge separation. Furthermore, in diagrams like Figure 2, where the "coordinate" used as the abscissa is discontinuous and of no meaningful physical denotation, the fact that the $e_{\mathrm{i}}$ add up to the total electronic energy makes their use preferable to using $\epsilon_{i}$,
methyl stabilization energy in the ethyl cation. From the highest occupied orbitals (the triply degenerate $1 t_{2}$ of methane and the doubly degenerate pairs $1 e_{u}$ and $l e_{g}$ ) the sum of MO stabilizations for reaction 1 is equal to approximately $184 \mathrm{kcal} / \mathrm{mol}$. The triply degenerate $1 \mathrm{t}_{2}$ of methane correlates with the $1 \mathrm{e}_{g}, 1 \mathrm{e}_{\mathfrak{u}}$, and $3 a_{1 g}$, and the corresponding stabilization energy for reaction 2 is $211 \mathrm{kcal} / \mathrm{mol}$, or $27 \mathrm{kcal} / \mathrm{mol} \mathrm{more}$. Of this, $12 \mathrm{kcal} / \mathrm{mole}$ is due to hyperconjugation (see below). By far the largest effect, however, is found in the difference in energy lowerings resulting from the MO's involving bonding with the carbon 2 s , and the major effect here ( $59 \mathrm{kcal} / \mathrm{mol}$ ) arises from CC $\sigma$ bonding. Inspection of Figure 1 and of the coefficients of the wave functions (not given here because of space limitations) reveals that the immediate effect of the decreased electron-electron repulsion about the positive center is to allow more electrons into the 2 s orbital of this carbon and to decrease the relative importance of the 2 p orbitals in bonding. Both the $\alpha$-hydrogens and their CH overlap regions become depopulated because the hydrogen 1 s orbitals are of much higher energy (ionization potential) than the carbon 2 s orbitals. ${ }^{15.17,18}$ Also, the CC overlap population changes little upon ionization, but because the methyl carbon has an orbital energetically comparable and hence ${ }^{26,32}$ available for mixing with the positive carbon 2 s (its own 2 s ) a greater percentage of that overlap is $2 \mathrm{~s}-2 \mathrm{~s}$. Since electrons in the 2 s orbitals see more of the carbon nuclei than do those in 2 p 's, the $2 \mathrm{~s}-2 \mathrm{~s}$ bonding is stronger than any bonding involving the 2 p or hydrogen orbitals.

A positively charged electron-deficient species such as $\mathbf{3}$ or $\mathbf{4}$ is stabilized not by intramolecular neutralization or dispersal of its excess charge but by optimal utilization of its electrons for filling the regions of high positive potential. The electron-deficient aspect is necessary because it makes electron repulsion less important and allows more electrons into smaller regions around the nuclei. The difference in the electronic structure between neutral and positively charged species is most clearly seen in the regrouping of electrons which takes place upon removal of an electron from the methyl radical to generate the methyl cation. A


geometry search yielded for the methyl radical a CH bond length of $1.077 \AA$, in agreement with other $a b$ initio calculations, ${ }^{33}$ and $1.091 \AA$ for the methyl cation. The shorter bond length of the radical compared to methane is easily explained either by the decreased nuclear repulsion around the carbon due to the lowering of its coordination number from 4 to 3 or by the change of its hybridization state.

The longer bond length in the cation is somewhat unexpected and warrants some further discussion. The effect of the decreased electron-electron repulsion is visible in the increase of the population of the carbon atomic orbitals. Removing an electron from the methyl radical and not counting the population of the $p_{\pi}$ orbital (which is 1 in the radical and 0 in the cation)
(33) V. Buss, unpublished STO-3G calculations; see also G. Herzberg, Proc. Roy. Soc., Ser. A, 262, 291 (1961).
actually result in an increase of negative charge on the carbon from +0.124 to -0.540 . This charge comes from the hydrogen atoms, which each lose about 0.19 electron, and from the carbon-hydrogen bonding region (the overlap population of which decreases from 0.734 to 0.556 ). Both the weaker bonds and the increased positive charge on the hydrogens account for the increased bond length.

The ethyl cation 4 has an extra stabilization over 3 because: (a) the electron deficiency of the positive center is transmitted into the methyl group resulting in an additional gain of nuclear-electron attraction, an effect which is not present in the parent ethane; (b) very efficient bonding involving the carbon 2 s orbitals is possible, while in the methyl cation the carbon 2 s cannot effectively mix with any of the available orbitals.

Reaction 2 is one of a number of reactions in which a CC bond between tetragonal carbons becomes shorter and more stable when one of the carbons becomes trigonal through loss of a substituent and all or part of the associated pair of bonding electrons. We can without further calculation apply the above analysis directly to such reactions, e.g., the dehydrogenation of propane to form propene


Here a mild electron deficiency is induced on the central carbon of propene by effectively dispersing a CH bond pair on the corresponding carbon of propane into the double bond and away from the methyl-methine CC bond. According to our analysis of the ethyl cation, this CC bond should become shorter and stronger, as is experimentally observed, and we thus account for the well-known stabilization of double bonds by alkyl substituents. ${ }^{34}$

Hyperconjugation vs. Bridging. By omitting the $p_{\pi}$ orbital on $C_{1}$ of 4 and repeating the calculation, we obtain structure 5 . In 5 , hyperconjugative resonance (4 $\leftrightarrow 4,{ }^{\prime}$ ), illustrated below, is completely suppressed.


The change in energy components for $\mathbf{4} \rightarrow \mathbf{5}$ is given in Table III. As expected, ${ }^{26}$ "second-order" hyperconiugation ${ }^{35}\left(\mathbf{4} \leftrightarrow 4,{ }^{\prime \prime} \mathbf{2} \leftrightarrow \mathbf{2}^{\prime}\right)$ is apparently not important,

in either $\mathbf{2}, \mathbf{4}$, or $\mathbf{5}$, for the $\pi_{y}$ overlap population in each of the three is net repulsive.
(34) See ref 26, Chapters 3 and 4.
(35) R. S. Mulliken, J. Chem. Phys., 7, 339 (1939); R. S. Mulliken, C. A. Rieke, and W. C. Brown, J. Amer. Chem. Soc., 63, 41 (1941).


5


6

Figure 3. Ab initio formal atomic charges and overlap populations (in parentheses) for $\pi$-localized ethyl cation and protonated ethylene.

The hyperconjugation energy in the ethyl cation is the energy change for the reaction $5 \rightarrow 4,{ }^{36}$ or, from Table III, $11.8 \mathrm{kcal} / \mathrm{mol}{ }^{38}$ This energy change is substantial, but not nearly as great as the calculated delocalization energy in the cyclopropenyl cation, 88 $\mathrm{kcal} / \mathrm{mol} .{ }^{4}$ This is as expected, because as hyperconjugation becomes important (indicated by resonance structure $4^{\prime}$ ) some of the CH binding energy in the methyl group must be lost. Hence, the methyl group CH overlap populations in the $\pi$-localized structure 5 are greater than those in 4 (Figures 1 and 3 ).

Analysis of the energy components for each of the molecular orbitals in $\mathbf{4}$ and 5 indicates that the changes in CH bonding are primarily responsible for the decrease in nuclear-electron attraction that takes place during the localization reaction (Table III). However, the lowered electron density and electron-electron repulsion around the positive carbon of 5 , relative to the positive carbon in 4 , allows some polarization of the CC $\sigma$ bond toward the electron-deficient center in 5, thus compensating in part for the loss in attractive potential. To be specific, about half the electrons donated in 4 to the positive carbon through the $\pi_{z}$ system reach the corresponding carbon in 5 through the $\sigma$ system, where proximity to the carbon nucleus and hence attractive potential is greater.

The energy component changes for a different delocalization process, the formation of the bridged ion 6, contrast sharply with the one just discussed (Table III). The nuclear-electron attraction strongly favors 6 over 4 and moderates the large increase in nuclear repulsions. Such moderation of nuclear repulsion by electronic energy is generally to be expected. As a molecular system becomes more compact, the attractive regions between nuclei become smaller and more attractive. The electrons in these regions are more strongly bound, but because the volume to which they are effectively confined has diminished, these electrons interact more with each other resulting in an increase of the $V_{\text {ee }}$ term. The bridge proton itself, as distinct from the orbitals centered on it, increases the stabilities of the molecular orbitals having large amounts of CC bonding character through its field effect (which is much stronger in the CC than in the CH overlap regions). Further stabilization of the CC bonding resulting from the $2 \mathrm{~s}-2 \mathrm{~s}$ overlap in the $2 \mathrm{a}_{1} \mathrm{MO}$ of 6

[^2]| No. | System ${ }^{\text {a }}$ | Atom | -...- Nuclear coordinates ${ }^{\text {b }}$ |  |  | -MMolecular orbital energies ${ }^{\text {b }}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $x$ | $y$ | $z$ | MO | $\epsilon_{i}$ | $e_{i}$ |
| 1 | Methane | C | 0.0 | 0.0 | 0.0 | $1 \mathrm{a}_{1}$ | -11.2266 | -15.5434 |
|  |  | H | 0.6904 | 0.0 | -1.9257 | $2 a_{1}$ | -0.9479 | -3.2046 |
|  |  | H | 0.6904 | -1.6911 | 0.9764 | $1 \mathrm{t}_{2}$ | -0.5436 | -2.6741 |
|  |  | H | 0.6904 | 1.6911 | 0.9764 |  |  |  |
|  |  | H | -2.0712 | 0.0 | 0.0 |  |  |  |
| 2 | Ethane | C | 0.0 | 0.0 | 0.0 | $1 a_{18}$ | -11.2439 | -16.7126 |
|  |  | C | -2.8989 | 0.0 | 0.0 | $1 \mathrm{a}_{2 \mathrm{u}}$ | -11.2434 | -16.7126 |
|  |  | H | 0.6904 | 0.0 | -1.9527 | $2 \mathrm{a}_{18}$ | -1.0286 | -4.5111 |
|  |  | H | 0.6904 | -1.6911 | 0.9764 | $2 \mathrm{a}_{2 \mathrm{u}}$ | -0.8435 | -4.0349 |
|  |  | H | 0.6904 | 1.6911 | 0.9764 | $1 \mathrm{e}_{\mathrm{u}}$ | -0.6068 | -3.7835 |
|  |  | H | -3.5893 | 0.0 | 1.9527 | $3 \mathrm{a}_{\text {lg }}$ | -0.4974 | -3.9941 |
|  |  | H | -3.5893 | 1.6911 | -0.9764 | $1 \mathrm{eg}_{8}$ | -0.4890 | -3.5747 |
|  |  | H | -3.5893 | -1.6911 | -0.9764 |  |  |  |
| 3 | Planar methyl cation | $\mathrm{C}^{+}$ | 0.0 | 0.0 | 0.0 | $1 a_{1}{ }^{\prime}$ | -11.7185 | -15.5565 |
|  |  | H | 2.0485 | 0.0 | 0.0 | $2 a_{1}{ }^{\prime}$ | -1.2907 | -3.2236 |
|  |  | H | -1.0242 | 1.7740 | 0.0 | $1 \mathrm{e}^{\prime}$ | -0.9581 | -2.8199 |
|  |  | H | -1.0242 | $-1.7740$ | 0.0 |  |  |  |
| 4 | $\begin{aligned} & \text { Planar ethyl } \\ & \text { cation, } \\ & 90,90 \end{aligned}$ | $\mathrm{C}^{+}$ | 0.0 | 0.0 | 0.0 | 1a | -11.7034 | -16.7529 |
|  |  | C | -2.7968 | 0.0 | 0.0 | 2a | -11.5305 | -16.7640 |
|  |  | H | 1.0242 | 1.7740 | 0.0 | 3a | -1.3457 | -4.6054 |
|  |  | H | 1.0242 | -1.7740 | 0.0 | 4 a | -1.1328 | -4.0795 |
|  |  | H | -3.4872 | 0.0 | 1.9527 | 1b | -0.9627 | -3.9100 |
|  |  | H | -3.4872 | 1.6911 | -0.9764 | 5 a | -0.8442 | -4.0201 |
|  |  | H | -3.4872 | -1.6911 | -0.9764 | 6a | -0.8350 | -3.8204 |
|  |  |  |  |  |  | 2b | -0.7892 | -3.7208 |
| 5 | Planar ethyl cation, $\pi$ localized |  | Coordinates as for 4 |  |  |  |  | -16.7641 |
|  |  | C |  |  |  | 2a | -11.5119 | -16.7547 |
|  |  | H |  |  |  | 3a | -1.3488 | -4.6008 |
|  |  | H |  |  |  | 4 a | -1.1375 | -4.0958 |
|  |  | H |  |  |  | 1b | -0.9762 | -3.9165 |
|  |  | H |  |  |  | 5a | -0.8452 | -4.0167 |
|  |  | H |  |  |  | 6a | -0.8194 | -3.7960 |
|  |  |  |  |  |  | 2b | -0.7812 | -3.7191 |
| 6 | Protonated ethylene | C | 1.3795 | 0.0 | 0.0 | $1 \mathrm{a}_{1}$ | -11.6389 | -16.7977 |
|  |  | C | -1.3795 | 0.0 | 0.0 | $1 \mathrm{~b}_{1}$ | -11.6381 | -16.7976 |
|  |  | H | 0.0 | 0.0 | 2. 1047 | $2 a_{1}$ | -1.3996 | -4.7363 |
|  |  | H | 2.4038 | 1.7740 | 0.0 | $2 \mathrm{~b}_{1}$ | -1.1169 | -4.0783 |
|  |  | H | 2.4038 | -1.7740 | 0.0 | $1 \mathrm{~b}_{2}$ | -0.9647 | - 3.9736 |
|  |  | H | -2.4038 | 1.7740 | 0.0 | $3 \mathrm{a}_{1}$ | -0.9023 | -3.9405 |
|  |  | H | -2.4038 | -1.7740 | 0.0 | $1 \mathrm{a}_{2}$ | -0.8350 | -3.7552 |
|  |  |  |  |  |  | $4 \mathrm{a}_{1}$ | -0.7890 | -3.8775 |

${ }^{a}$ For values of internal coordinates see Table I . ${ }^{b}$ Coordinates in bohrs ( $1 \mathrm{bohr}=0.52917 \AA$ ); energies in hartrees $(1$ hartree $=627.502$ $\mathrm{kcal} / \mathrm{mol}$ ).
arises because the electron deficiency in 6 is equally shared between both carbons. That is, the $2 a_{1}$ is not centered on (or polarized toward) either carbon, as is the case in 4, but is rather spread over both, which reduces T and increases the overlap population and $V_{\text {ne }}$ relative to the 3 a MO of 4 . Of course, movement of the hydrogen in 4 ( $\mathrm{H}_{\mathrm{s}}$ in Figure 1) to the bridge position of 6 entails loss of some CH binding energy. In addition the 2 p orbitals lose the CC bonding character they have in the 5 a of 4 and are relegated to bonding in the ethylenic CH bonds in the $3 \mathrm{a}_{1}$ of 6 . Both of these result in the destabilization of the $3 a_{1}$-the only major destabilization indicated by the $e_{i}$ of an MO resulting from the process $\mathbf{4} \rightarrow \mathbf{6}$ (see Table IV).

As Table III indicates, the net effect of the changes in electronic structure and energy in $\mathbf{4 \rightarrow 6}$ is not quite enough to overcome the increase in nuclear repulsions, and so 4 is favored by $9.0 \mathrm{kcal} / \mathrm{mol}$. We expect that this energy difference would not change significantly if we were to make the refinements necessary to push our calculations to the Hartree-Fock limit. Furthermore, we do not expect the correlation energies for the two ions to be very different, for reasons given earlier. ${ }^{2}$ In fact, the Hollister-Sinanoglu ${ }^{40}$ method predicts a
correlation energy change of $0.2 \mathrm{kcal} / \mathrm{mol}$ (i.e., negligible for our purposes) for the $4 \rightarrow 6$ isomerization.

It has been stated (e.g., by Dewar ${ }^{26}$ ) that molecular orbital methods will always overestimate delocalization energies because these methods do not take into account correlation effects. That is, by correlating their motions, electrons can stay out of each other's way and thus better avoid repulsive interactions. Since in a delocalized species the electrons should be farther apart than in a corresponding localized isomer, it is argued that correlation should be a smaller effect for the delocalized form, i.e., should favor the more crowded structure. We now have in hand, however, near-Hartree-Fock wave functions for the species involved in the delocalization processes in Table V. Obviously, if we use the size of the electron-electron interaction energies as measures of roughly how much might be gained from electron correlations, then the delocalized forms, except the $\pi$-localized ethyl cation, should be favored by correlation effects. As we have argued, ${ }^{2,4}$ these effects should be small for the reactions noted above. On the other hand, semiempirical MO
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Table V

| $\mathrm{H}_{\mathrm{H}}^{-}$ | $\rightarrow$ | $\stackrel{H}{+}$ | 27 | a |
| :---: | :---: | :---: | :---: | :---: |
|  | $\rightarrow$ |  | 118 | a |
|  | $\rightarrow$ |  | -40 | This work |
|  | $\rightarrow$ |  | 202 | This work |
| $\mathrm{H}_{2} \mathrm{C}=\mathrm{CH}^{+}$ | $\longrightarrow$ |  | 110 | $b$ |

[^3]schemes do in general overestimate resonance effects. ${ }^{2,7,9,10,35,37}$ This is due to inadequate treatment of two-electron effects, especially electron-electron repulsions, rather than to neglect of correlation energies.

## Conclusions

The stabilities of alkyl cations are not dominated by their ability to neutralize or disperse positive charge, but by optimum utilization of the most stable orbitals. The electron-deficient nature of these molecules is important because it lowers electron-electron repulsion, thus allowing electrons to crowd close together at places of high positive potential. As a result of the greater ability of a methyl carbon than of a hydrogen substituent to take advantage of this lowered repulsion (by forming strong bonds with the cationic carbon through involvement of the 2 s orbitals), the ethyl cation is more easily formed from its neutral precursor than is the methyl cation.

# Stable Carbocations. CXII. Preferential Formation of the Bicyclo[3.3.0]-1-octyl Cation from Bicyclooctyl Precursors and Its Rearrangement to the 2-Methylnorbornyl Cation ${ }^{1}$ 

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#### Abstract

The bicylo[3.3.0]-1-octyl cation (I) is formed from all bicyclooctyl precursors in $\mathrm{FSO}_{3} \mathrm{H}-\mathrm{SbF}_{5}-\mathrm{SO}_{2} \mathrm{ClF}$ or in $\mathrm{SbF}_{5}-\mathrm{SO}_{2} \mathrm{ClF}$ solutions at $-78^{\circ}$. When I is heated to $0^{\circ}$, it irreversibly rearranges to 2-methylnorbornyl cation.


I$n$ the solvolytic studies of bicyclo-2-octyl derivatives, ${ }^{2-7}$ it was shown that two distinct carbocation intermediates are involved, the asymmetrical nonclassical ion II formed from exo-bicyclo[3.2.1]-2-octyl isomers and the symmetrical nonclassical ion III formed from endo-bicyclo[3.2.1]-2-octyl isomers. Similar observations were made in the solvolyses of both exo-and endo-2-norbornecarbinyl ${ }^{8-12}$ and 7 -norbornanecarbinyl systems. ${ }^{13,14}$

The solvolytic products involve both bicyclo[2.2.2]and [3.2.1]octyl structures. No further rearrangement
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I

II

III
products were observed. However, when initial rearrangement can produce bicyclo[3.3.0]octyl cations, bicyclo[3.3.0]octyl products are dominant. ${ }^{15-20}$ In the aluminum bromide catalyzed isomerization of a series of bicyclooctanes, the following stability order is revealed. ${ }^{21}$


[^4]
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